PROPERTIES OF GAS FILTRATION IN A CENTRIFUGAL FIELD

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Data are presented on theoretical and experimental studies of the rate of gas filtration through rotating cylindrical bodies.

The important difference in the resistance of rotating and stationary diffusers to the flow of a gas stream was shown in [1, 2]. These differences are due to the presence of a pseudogravitational field and depend on the nature of the gas, the nature of the porous material, and the direction of gas flow [3, 4]. A commercial-type model was taken as the model of the porous material. It is known that in such a model the material is considered to be penetrated by n pores having an effective diameter d. The transmissive capacity of the n pores is equal to the transmissive capacity of the porous material. If inertial losses are neglected, the flow through an individual capillary is described by the expression

$$Q = \frac{1}{8\eta}\left(1 + \frac{8(2-f)}{f}\cdot\frac{\lambda}{d}\right)\frac{\pi d^4}{16RT}\,p\,\frac{\Delta p}{\Delta x}. \quad (1)$$

In Trebin's report [5] it was shown experimentally that with flow of this nature the effective pore diameter equals

$$d = 4\sqrt{\frac{2k_f}{\sigma}}. \quad (2)$$

Equation (2) can also be obtained theoretically on the basis of Eq. (1). The density [kmole/m^2•sec] of a gas stream passing through n capillaries will equal

$$j = \frac{\pi d^2}{32\eta RT}\left(1 + \frac{8(2-f)}{f}\cdot\frac{\lambda}{d}\right)p\,\frac{\Delta p}{\Delta x}. \quad (3)$$

Diffusers of cylindrical shape are most often encountered in the analysis of the filtration of a gas stream in a centrifugal field. For such diffusers the stream density at the entrance is connected with the density at a distance x from the axis by the equation

$$j_x = j_0\,\frac{r}{x}. \quad (4)$$

Equation (4), being a consequence of the continuity equation, will be valid for an isotropic diffuser. In the general case it follows from the continuity equations that

$$j_0 S' = j_x S'',$$

from which we have

$$j_x = j_0\,\frac{\sigma'}{\sigma''}\cdot\frac{r}{x}, \quad (5)$$

which converts to (4) when the medium is isotropic ($\sigma'' = \sigma'$). If the flow obeys Darcy's law (η = const, Kn → 0), then from (3) for an individual capillary we have ($\lambda \ll d$)

$$d = \left(\frac{32 j_0 r\eta\mu}{\rho\Delta p}\right)^{1/2}. \quad (6)$$

The expression $j_0\mu$ [(kmole/m^2•sec)•(kg/kmole] is the mass flow of gas through a unit area in a unit time. In such a case $j_0\mu/\rho$ [m/sec] is the velocity v of this flow. Therefore,

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$$d=4\left(\frac{2v\eta r}{\Delta p}\right)^{1/2}. \quad (7)$$

The velocity v which enters into (7) is the average velocity of the particles passing through the pore. It is equal to the ratio of the volumetric flow V to the area of the pores: $V/S = v_m/\sigma$. Since on the basis of the Kármán–Kozen' equation [6] the mean linear flow velocity v_m normalized to the cross-sectional area of the body equals $(k_f/\eta)\cdot(\Delta p/r)$, we obtain for the effective diameter

$$d=4\sqrt{\frac{2k_f}{\sigma}}.$$

The effective diameter determined by Eq. (2) was used to test the equations of pressure distribution and gas flow rate in different modes of filtration in a flat stationary membrane and good agreement between the experimental and theoretical data was obtained [7]. Thus, a commercial-type model is suitable for use in research because of the small number of easily measured experimental quantities which characterize it.

When a diffuser is placed in a centrifugal field a pressure difference $\Delta p_2 = Kxp\Delta x$ due to the presence of the centrifugal field [1] will exist in addition to the difference Δp in pressures on the two sides of the diffuser due to the presence of the resistance of the porous material. The pressure distribution equation in accordance with Darcy's law is obtained in [1].

In a viscous mode with slippage the second term cannot be neglected and the expression for the stream density when the flow is isothermal will be

$$j=B_1p\frac{\Delta p}{\Delta x}+B_2\frac{\Delta p}{\Delta x}. \quad (8)$$

The coefficients

$$B_1=\frac{3\pi^{3/2}d^3(2-f)\sqrt{RT}}{64\sqrt{2\mu}} \text{ and } B_2=\frac{3\pi^{5/2}d_e^2\sqrt{R}\,d^4}{256k\sqrt{\mu T}}$$

depend on the nature of the gas and the effective diameter of the pores. For oxygen with $d = 10^{-5}$ m and $T = 300°K$ we have $B_1 = 5\cdot10^{-14}$ [$m\cdot sec^3\cdot kmole\cdot kg^{-2}$] and $B_2 = 6.3\cdot10^{-10}$ [$kmole\cdot sec\cdot kg^{-1}$]. For a stationary diffuser

$$pdp+Bdp=F\frac{dx}{x},$$

from which we have the following expression for the pressure distribution:

$$p^2+2Bp=p_0^2+2Bp_0+2F\ln\frac{x}{r}. \quad (9)$$

Whereas the coefficient $B = B_1/B_2$ [$kg\cdot m^{-1}\cdot sec^{-2}$] depends on the nature of the gas and the diameter of the diffuser pores, the coefficient $F = 1/B_2$ [$kg^2\cdot m^{-2}\cdot sec^{-4}$] depends, in addition, on the conditions at the entrance, particularly on the density of the entering stream. For this reason the coefficient F is determined in each individual case from the known parameters of the diffuser and the gas flow rate.

In the presence of rotation Eq. (9) takes the form

$$dp=\frac{Fdx}{B_1xp+B_2x}+Kxpdx. \quad (10)$$

For the flow density in the molecular mode ($Kn\to\infty$) we have

$$j=\frac{2\sqrt{2}\,d}{\sqrt{\pi\mu RT}}\cdot\frac{\Delta p}{\Delta x}=A\frac{\Delta p}{\Delta x}. \quad (11)$$

If the rotor rotates by analogy with the previous case we obtain the following equation for the pressure distribution:

$$p=p_0\exp[-K(r^2-x^2)]-B\exp(Kx^2)\int_{x}^{r}\frac{\exp(-Kx^2)}{x}dx. \quad (12)$$

In order to obtain the equation for the distribution in a stationary rotor it is sufficient to take K as equal to zero.

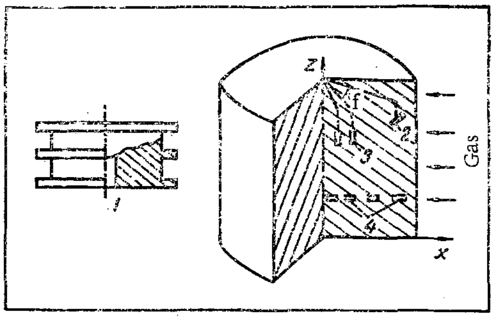

Fig. 1. Diagram of arrangement of pickups for measuring the velocity of gas streams within the rotor: 1) commutator; 2) pickup for measuring azimuthal velocity; 3) pickup for measuring velocity of axial streams; 4) pickup for measuring dependence of velocity on variation in the distance from the rotation axis.

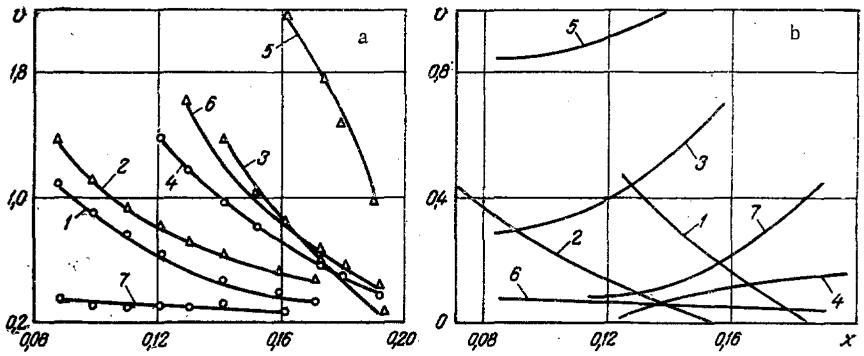

Fig. 2. Dependence of gas flow velocity along radius of rotor in the viscous mode (a) and in the molecular mode and the viscous mode with slippage (b). For a: 1) $p_0 = 10^5$; 2–7) $p_0 = 7 \cdot 10^4$; 1) $B = 4 \cdot 10^8$; 2, 3,5) $B = 2 \cdot 10^8$; 4) $B = 10^8$; 6,7) $B = 5 \cdot 10^7$; 1,4,5,6) $K = 0$; 2,3,7) $K = 10$; for b: 1-5) $p_0 = 4 \cdot 10^4$; 6,7) $p_0 = 10^5$; 1-3) $B = 2 \cdot 10^8$; 4,5) $B = 5 \cdot 10^8$; 6,7) $B = 5 \cdot 10^7$; 1,6) $K = 0$; 2,4) $K = 10$; 3,5,7) $K = 90$.

To test all these equations it was necessary to measure the pressure at different points of stationary and rotating diffusers. Experimental studies for stationary diffusers are described in [2, 3] and gave good agreement with the calculated data. But in a study of the pressure distribution in a rotating rotor the tests are associated with a number of experimental difficulties, connected, in particular, with the determination of the pressure at different distances from the axis of a rapidly rotating rotor. To overcome these difficulties the velocity of the gas streams at different points of the rotor was measured. The semiconductor pickups for measuring the flow velocity have a small size and essentially do not affect the flow velocity. Moreover, because of the small mass they are not subject to the effect of large forces on the part of the centrifugal field. A diagram of the arrangement of the pickups is shown in Fig. 1. We also intended to study the presence of azimuthal flows in the measurements. For this the pickups were placed in gas-tight cylinders without ends and arranged both along the radius and parallel to the axis of the rotor and to its lateral surface. The pickups located along the radius measured the filtration velocity, while the others measured the axial and azimuthal velocities. The leads from the pickups were connected to a commutator. The pickup resistance, which depends on the velocity of the gas stream, was measured with a six-armed electrical bridge [8]. The errors in the measurements of the flow velocity did not exceed 0.5% of the measured quantity. The pickups were preliminarily graduated and subsequently the graduation data were used.

In order to compare the experimental data obtained with theoretical data we used the Navier—Stokes equation, which in the case of a one-dimensional mode of flow takes the form

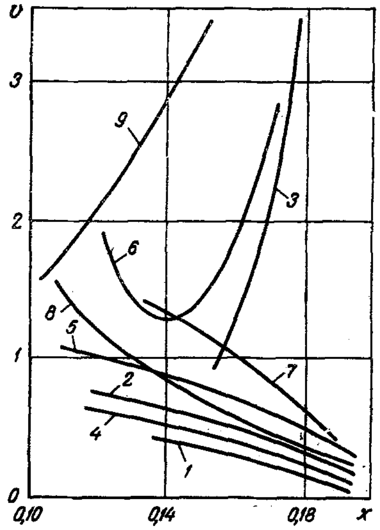

Fig. 3. Dependence of gas flow velocity along radius of a porous rotor at different rpms ($p_0 = 4 \cdot 10^4$): 1-3) $B = 4 \cdot 10^7$; 4-6) $B = 5 \cdot 10^7$; 7-9) $B = 10^8$; 1,4, 7) K = 0; 2,5,8) K = 10; 3,6,9) K = 90.

$$\frac{\eta RT}{p\mu}\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{1}{x}\cdot\frac{\partial v_x}{\partial x}-\frac{v_x}{x^2}\right)-v_x\frac{\partial v_x}{\partial x}=\frac{RT}{p\mu}\cdot\frac{\partial p}{\partial x}. \tag{13}$$

By substituting into (13) the values of the pressures given by Eqs. (9), (10), (12), and others we found the filtration velocity at a given point. In solving the equations we used the method of finite differences. The results were processed on a Minsk-22 computer. To test the convergence of the solutions we varied the step of the computation and compared the data for different steps. The results of the studies are presented in Figs. 2 and 3. The experimental studies were performed on a porous specimen with an effective pore diameter of $1.7 \cdot 10^{-6}$ m and $\sigma = 0.37$. The essential dependence of the flow velocity on the parameter B [N^2/m^4], which characterizes the gas stream and the porous material, and on the parameter K [m^{-2}], which characterizes the magnitude of the centrifugal field, is seen from the figures. At a low rpm the filtration velocity increases toward the rotor axis. An increase in the rpm leads to an increase in the velocity toward the outer wall (Fig. 2b). The presence of elastic and viscous forces can lead to inflection of the curves (Fig. 3). This once again confirms the nonlinear connection between the stress tensor and the deformation rate tensor. This connection was discovered experimentally for gas flow between disks [9] and was noticed in a porous rotor [3].

The comparison of the experimental and theoretical data for the velocity of the gas streams not only makes it possible to judge the nature of the flow within a porous rotating rotor, but also to confirm the validity of the equation for the pressure distribution.

NOTATION

Q, amount of gas flow, mole/sec; η, dynamic viscosity coefficient; f, reflection coefficient; λ, mean free path of gas molecules; d, effective pore diameter; R, universal gas constant; T, absolute temperature; p, pressure; Pa, Δp, pressure difference; x, coordinates of point under consideration; k_f, filtration coefficient; σ, porosity coefficient; j, gas stream density; j_0, stream density at entrance to diffuser, kmole/m^2•sec; r, outer radius of diffuser; S, area of pores at a distance x from axis; σ', σ'', porosity coefficients at the same points; μ, molecular mass of gas; ρ, gas density; v, gas stream velocity, m/sec; $K = m\omega^2/kT$, m^{-2}, and A_1, B_1, F, $B = j_0 r/A$, constants; d_e, effective diameter of molecules; k, Boltzmann's constant; m, mass of molecules; ω, angular velocity of rotation; Kn, Knudsen number.

LITERATURE CITED

1. P. Ya. Nedviga and S. I. Kulikovskii, Inzh.-Fiz. Zh., 16, No. 5 (1969).
2. S. I. Kulikovskii, Izv. Vyssh. Uchebn. Zaved. SSSR, Fiz., No. 4 (1972).
3. S. I. Kulikovskii and G. Ya. Gridasov, in: Urgent Problems of Solid State Physics [in Russian], Kiev (1973), p. 78.
4. S. I. Kulikovskii and P. Ya. Nedviga, in: Urgent Problems of Solid State Physics [in Russian], Kiev (1970), p. 118.
5. G. F. Trebin, Filtration of Liquids and Gases in Porous Media [in Russian], Gostoptekhizdat (1959).

6. A. V. Lykov, Heat and Mass Exchange [in Russian], Moscow (1972).
7. P. A. Novikov and V. A. Eremenko, Inzh.-Fiz. Zh., 23, No. 5 (1972).
8. P. Ya. Nedviga, G. Ya. Gridasov, and S. I. Kulikovskii, Priborostroenie, No. 15 (1973).
9. A. V. Lykov, Z. P. Shul'man, et al., Inzh.-Fiz. Zh., 22, No. 2 (1972).

PERFORMANCE OF FILM COOLING BY INJECTION FROM A POROUS SECTION

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Results are presented on film cooling by homogeneous injection into a turbulent boundary layer through porous inserts of various lengths along the main flow direction.

It is usual [1] to use the temperature of a thermally insulated wall in conjunction with the ordinary equation of heat transfer in calculating the parameters of elements working under conditions of severe heat loading and cooled by gas injection. The performance parameter for the film cooling may serve to characterize the adiabatic wall temperature, and this can be defined as

$$\eta = \frac{T_0 - T_{a.w}}{T_0 - T_1}\,. \tag{1}$$

It is usual to apply correction factors to this performance parameter in order to incorporate the effects of factors such as the physicochemical characteristics of the two flows, the deviation from isothermal conditions, the compressibility, and so on, all of which substantially affect the heat transfer.

Results have been given [2-5] from measurements on the cooling performance behind porous elements used in such injection; however, the results correspond in the main to conditions under which there is very extensive injection through the porous surface, and hence the main flow is repelled from the surface.

On the other hand, considerable interest attaches to the use of such porous sections when the injection parameters are medium or small ($m \leq 10^{-2}$).

Here we report measurements on the performance of gas injection through porous sections of various length along the flow direction for a wide range of injection parameters.

The system was built around a rectangular wind tunnel; Fig. 1 shows the working section. The working channel had a cross section of 90 × 90 mm and was made of lucite. The lower wall of the working part consisted of the model. The porous plate block 6 was of sealed construction, and one wall bore the porous plate with Chromel—Copel thermocouples emerging on the hot surface, which were made of wire of diameter 0.2 mm. The porous plates were made by rolling stainless steel wire mesh. The effective porosity of such plates was about 30%.

We used three such porous blocks, which were similar in design but differed in length along the flow direction: L = 380, 190, and 40 mm.

The porous section was preceded by a water-cooled part 5; this served to maintain a constant surface temperature on the impermeable part and also a largely constant temperature on the porous section.

The thermally insulated plastic plate 7 was 30 mm thick and of length 1 m; temperature transducers were set up on this surface with a pitch of 10 mm, and these consisted of small copper sections fitted with the Chromel—Copel thermocouples 8.

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